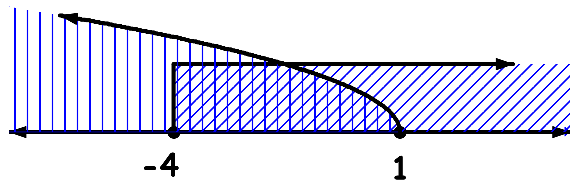
Math 0350 Practice Test **Unit Two**
Chapters 9 and10 part A **Martin-Gay 4th edition**

Solve the compound inequality. Graph the solution set and write it in interval notation.

1. $3x+1<4$ and $2x+4\geq -4$

Solution: The "and" in the problem means that both equations have to be true simultaneously.

$$\begin{array}{lcl} 3x+1<4 & \textit{and} & 2x+4\geq -4 \\ 3x<4-1 & & 2x\geq -4-4 \\ 3x<4 & & 2x\geq -8 \\ x<1 & & x\geq -4 \end{array}$$

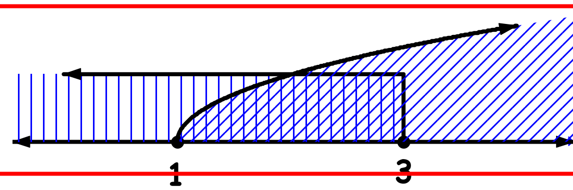


-4 is a valid solution (because of the equal sign), so we use brackets at -4. 1 is not included, so we use parentheses at 1. Both inequalities are true for x in the interval $\boxed{[-4,1)}$.

2. $5x-3>2$ or $-2x\geq -6$

Solution: The "or" in the problem means that the solution is where either one of the inequalities is true.

$$\begin{array}{lcl} 5x-3>2 & \textit{or} & -2x\geq -6 \\ 5x>2+3 & & x\leq 3 \\ 5x>5 & & \\ x>1 & & \end{array}$$

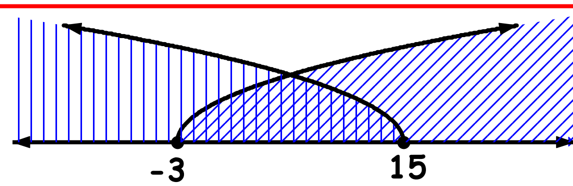


3 is a valid solution (because of the equal sign), so we use brackets at 3. 1 is not included, so we use parentheses at 1. One or both inequalities are true for x in the interval $\boxed{(-\infty,\infty)}$.

3. $-2<\dfrac{x-3}{3}<4$

Solution: Solve both inequalities simultaneously. The solution is where both inequalities are true.

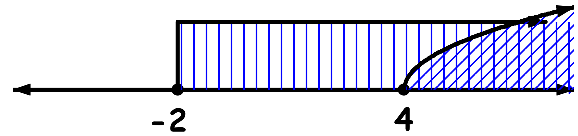
$$-2<\frac{x-3}{3}<4$$
$$-2(3)<\frac{x-3}{3}(3)<4(3)$$
$$-6<x-3<12$$
$$-6+3<x<12+3$$
$$-3<x<15$$



Both inequalities are true for x in the interval $(-3,15)$.

4. $x > 4$ or $x \geq -2$

Solution: The "or" in the problem means that the solution is where either one of the inequalities is true. -2 is a valid solution (because of the equal sign), so we use brackets at -2. One or both inequalities are true for x in the interval $\boxed{[-2,\infty)}$



Solve each equation or inequality. Use interval notation when appropriate.

5. $|3x+6|-7=8$

Solution: Solve for the absolute value first: $|3x+6|=15$.

Divide the problem into 2 problems: $3x+6=15$ *and* $3x+6=-15$

Solve. There are 2 solutions for most of the absolute value equations:

$$3x+6=15$$
$$3x=15-6$$
$$3x=9$$
$$\boxed{x=3}$$

and

$$3x+6=-15$$
$$3x=-15-6$$
$$3x=-21$$
$$\boxed{x=-7}$$

6. $|4x-7|=-5$

Solution: Absolute value represents a distance, so this problem doesn't have a solution; the distance cannot be a negative number.

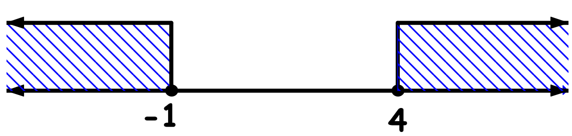
7. $|3-2x| \geq 5$

Solution: Absolute value represents a distance, so this problem is $3-2x=?$ so $|3-2x| \geq 5$. This inequality is true when

$$3-2x \geq 5$$
$$-2x \geq 5-3$$
$$-2x \geq 2$$
$$x \leq -1$$

and

$$3-2x \leq -5$$
$$-2x \leq -5-3$$
$$-2x \leq -8$$
$$x \leq 4$$



In interval notation: $(-\infty,-1] \cup [4,\infty)$

9. $|4-5x|>-6$

Solution: Absolute value represents a distance, so this problem is $4-5x=?$ so $|4-5x|>-6$. This inequality is always true, because the distance is always greater or equal to 0. In interval notation: $\boxed{(-\infty,\infty)}$

10. $|2-3x|=|x+2|$

Solution: Divide the problem into 2 problems: $2-3x=+(x+2)$ *and* $2-3x=-(x+2)$

Solve. There are 2 solutions for most of the absolute value equations:

$2-3x=+(x+2)$		$2-3x=-(x+2)$
$2-3x=x+2$		$2-3x=-x-2$
$-3x=x$	and	$-3x=-x-2-2$
$-3x-x=0$		$-3x+x=-4$
$-4x=0$		$-2x=-4$
$\boxed{x=0}$		$\boxed{x=2}$

11. $|x+3|+2<5$

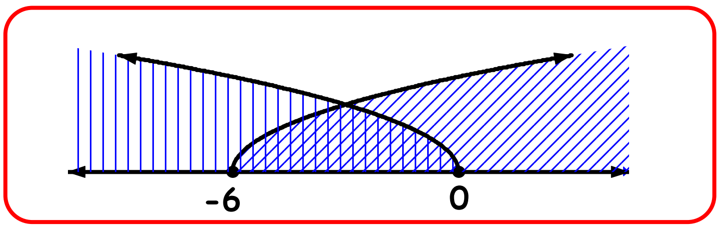
Solution: Solve for the absolute value first: $|x+3|<5-2;\ \ |x+3|<3$.

Absolute value represents a distance, so this problem is $x+3=?$ so $|x+3|<3$. This inequality is true when

$-3<x+3<3$

$-3-3<x<3-3$

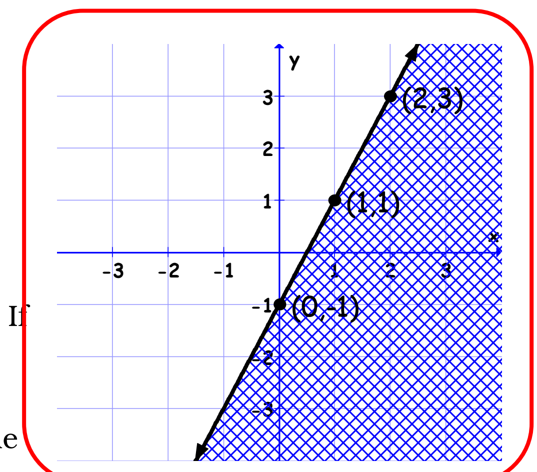
$-6<x<0$



In interval notation: $\boxed{(-6,0)}$

Graph each inequality

12. $y\le 2x-1$



Solution: Step I: Graph the line $y=2x-1$ using any of the methods you know.

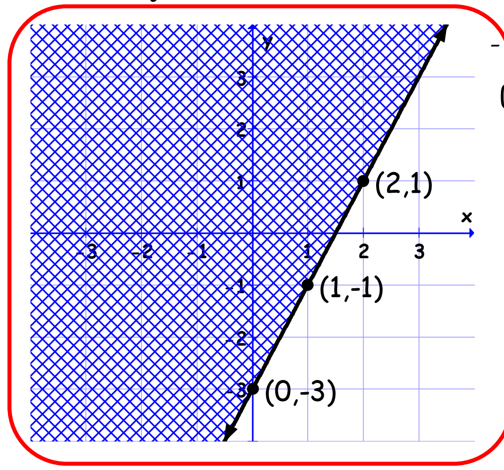
Step II: the solution will be either above the line or below the line. We can check which area is the solution by plugging a random point that does not belong to the line in the inequality: If we select the point (0, 0): $0\le 2(0)-1$ *wrong*!

The side that contain the point (0, 0) is not a solution. We shade the other side. Make sure that you graph a bold line, because the line itself is a solution.

13. $4x - 2y < 6$

Solution: Step I: Graph the line $4x - 2y = 6$ using any of the methods you know.

If we plug values for x = 0, 1, 2, we get the values for y = -3, -1, 1, so points from the line are the points: (0, -3); (1, -1); and (2, 1).



Step II: the solution will be either above the line or below the line.

We can check which area is the solution by plugging a random point that does not belong to the line in the inequality: If we select the point (0, 0):

$$4x - 2y < 6$$

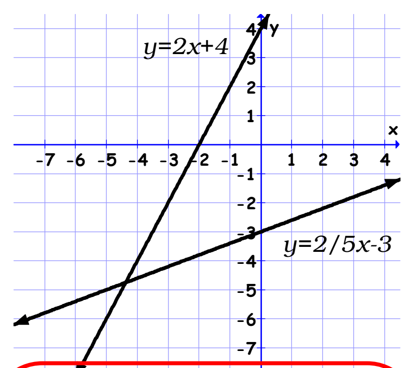
$$4(0) - 2(0) < 6 \quad correct!$$

The side containing the point (0, 0) is a solution. We shade the side containing the point (0, 0). Make sure you draw a dotted line, because $4x - 2y < 6$ is a strong inequality which tells us that the points from the line are not solution.

Graph the solution of the following systems of linear inequalities

14. $\begin{cases} 2x - y \geq -4 \\ y > \dfrac{2}{5}x - 3 \end{cases}$

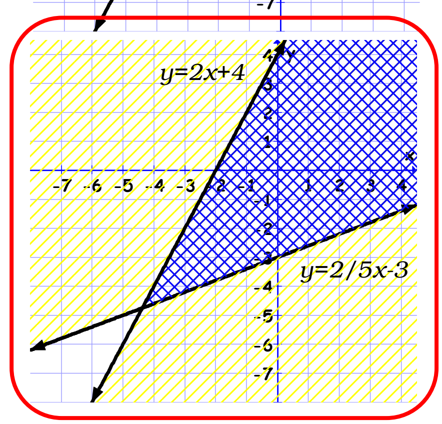
Solution: The first inequality has to be graphed with a bold line. The second inequality is a strong inequality, so it has to be graphed with a dotted line:



Perform a point check. Take a point from INSIDE of the areas and plug in in the INEQUALITIES! I will pick the point $(0,0)$

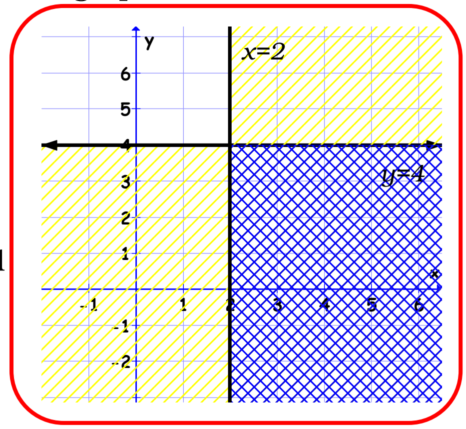
$$\begin{cases} 2(0) - 0 \geq -4 & \color{red}{correct} \\ 0 > -3 & \color{red}{correct} \end{cases} \Rightarrow$$ The point $(0,0)$ belongs to the solution!

The solution is:



15, $\begin{cases} y < 4 \\ x \geq 2 \end{cases}$

Solution: The first inequality is a strong inequality, so it has to be graphed with a dotted line; the second inequality has to be graphed with a bold line.
We can either perform a point check or we can shade the areas as described: $y < 4$ are all the points that have y value less than 4, so we shade below the line $y = 4$.
$x \geq 2$ are all the points that have x coordinate greater or equal to 2, so we shade to the right of the line $x = 2$.
The areas overlap in the lower right corner of the graph.



Write each of the following in simple radical form. Assume that all variable expressions represent positive numbers.

16. $\sqrt[3]{64x^9y^6}$

Solution: $\sqrt[3]{64x^9y^6} = \sqrt[3]{64}\ \sqrt[3]{x^9}\ \sqrt[3]{y^6} = 4\ x^{\frac{9}{3}}\ y^{\frac{6}{3}} = \boxed{4x^3y^2}$

17. $-\sqrt{49x^3y^{16}}$

Solution: $-\sqrt{49x^3y^{16}} = -\sqrt{49}\sqrt{x^3}\sqrt{y^{16}} = -7\ \sqrt{x^2}\ \sqrt{x}\ \ y^{\frac{16}{2}} = \boxed{-7\,x\,y^8\sqrt{x}}$

18. $\sqrt[4]{16x^{10}y^5}$

Solution: $\sqrt[4]{16x^{10}y^5} = \sqrt[4]{16}\ \sqrt[4]{x^{10}}\ \sqrt[4]{y^5} = 2\ \sqrt[4]{x^8}\ \sqrt[4]{x^2}\ \sqrt[4]{y^4}\ \sqrt[4]{y} = 2\ x^{\frac{8}{4}}\ y^{\frac{4}{4}}\ \sqrt[4]{x^2}\ \sqrt[4]{y} = \boxed{2\ x^2y\ \ \sqrt[4]{x^2y}}$

19. $\sqrt[5]{(x-2)^5}$

Solution: $\sqrt[5]{(x-2)^5} = (x-2)^{\frac{5}{5}} = (x-2)^1 = \boxed{x-2}$

20. $\sqrt[3]{\dfrac{54x^{13}y^5}{2x^4y^2}}$

Solution: $\sqrt[3]{\dfrac{54x^{13}y^5}{2x^4y^2}} = \sqrt[3]{27x^{13-4}y^{5-2}} = \sqrt[3]{27x^9y^3} = \sqrt[3]{27}\sqrt[3]{x^9}\sqrt[3]{y^3} = 3x^{\frac{9}{3}}y^{\frac{3}{3}} = \boxed{3x^3y}$

Simplify each expression. Assume that all variables represent positive numbers. Exponents in the final answer should be positive.

21. $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{-1}{3}}}\right)^6$

Solution: $\left(\dfrac{x^{\frac{2}{3}}}{y^{\frac{-1}{3}}}\right)^6 = \dfrac{\left(x^{\frac{2}{3}}\right)^6}{\left(y^{\frac{-1}{3}}\right)^6} = \dfrac{x^{\frac{2}{3}\cdot 6}}{y^{\frac{-1}{3}\cdot 6}} = \dfrac{x^4}{y^{-2}} = \boxed{x^4y^2}$

22. $\left(\dfrac{x^{\frac{1}{2}}}{y}\right)^{-2}$

Solution: $\left(\dfrac{x^{\frac{1}{2}}}{y}\right)^{-2} = \dfrac{\left(x^{\frac{1}{2}}\right)^{-2}}{\left(y^1\right)^{-2}} = \dfrac{x^{\frac{1}{2}(-2)}}{y^{1\cdot(-2)}} = \dfrac{x^{-1}}{y^{-2}} = \boxed{\dfrac{y^2}{x}}$

23. $a^{\frac{2}{3}}(a^{\frac{1}{3}} - 2a^{\frac{4}{3}})$

Solution: $a^{\frac{2}{3}}(a^{\frac{1}{3}} - 2a^{\frac{4}{3}}) = a^{\frac{2}{3}}(a^{\frac{1}{3}}) + a^{\frac{2}{3}}(-2a^{\frac{4}{3}}) = a^{\frac{2}{3}+\frac{1}{3}} - 2a^{\frac{2}{3}+\frac{4}{3}} = a^{\frac{3}{3}} - 2a^{\frac{6}{3}} = \boxed{a - 2a^2}$

Perform the indicated operations. Assume that all variables represent positive numbers.

24. $\sqrt[3]{25x^2y^4}\cdot\sqrt[3]{5x^7y^8}$

Solution: $\sqrt[3]{25x^2y^4}\cdot\sqrt[3]{5x^7y^8} = \sqrt[3]{25x^2y^4 5x^7y^8} = \sqrt[3]{125x^{2+7}y^{4+8}} = \sqrt[3]{5^3x^9y^{12}} = 5\,x^{\frac{9}{3}}\,y^{\frac{12}{3}} = \boxed{5\,x^3\,y^4}$

25. $\dfrac{6\sqrt{a^5b}}{\sqrt{4a^2b^3}}$

Solution: $\dfrac{6\sqrt{a^5b}}{\sqrt{4a^2b^3}} = 6\sqrt{\dfrac{a^5b}{4a^2b^3}} = 6\sqrt{\dfrac{a^{5-2}}{4b^{3-1}}} = 6\sqrt{\dfrac{a^3}{4b^2}} = \dfrac{6\sqrt{a^3}}{\sqrt{4b^2}} = \dfrac{6\sqrt{a^2}\sqrt{a}}{\sqrt{4}\sqrt{b^2}} = \dfrac{6a\sqrt{a}}{2b} = \boxed{\dfrac{3a\sqrt{a}}{b}}$

26. $3\sqrt{32x^2} + 5x\sqrt{8}$

Solution: $3\sqrt{32x^2} + 5x\sqrt{8} = 3\sqrt{16(2)x^2} + 5x\sqrt{4(2)} = 3\sqrt{16}\sqrt{2}\sqrt{x^2} + 5x\sqrt{4}\sqrt{2} =$

$= 3\cdot 4x\sqrt{2} + 5x\cdot 2\sqrt{2} = 12x\sqrt{2} + 10x\sqrt{2} = \boxed{22x\sqrt{2}}$

27. $(2-3\sqrt{3})(2+3\sqrt{3})$

Solution:

$$(2-3\sqrt{3})(2+3\sqrt{3})=2\cdot 2+2\cdot 3\sqrt{3}-3\sqrt{3}\cdot 2-3\sqrt{3}\cdot 3\sqrt{3}=4+6\sqrt{3}-6\sqrt{3}-9\left(\sqrt{3}\right)^2=4-9\cdot 3=4-27\boxed{=-23}$$

28. $(\sqrt{7}+\sqrt{2})^2$

Solution:

$$(\sqrt{7}+\sqrt{2})^2=(\sqrt{7}+\sqrt{2})(\sqrt{7}+\sqrt{2})=\sqrt{7}\sqrt{7}+\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}+\sqrt{2}\sqrt{2}=7+\sqrt{14}+\sqrt{14}+2=\boxed{9+2\sqrt{14}}$$

29. $\sqrt[3]{54x^4}+4x\ \sqrt[3]{16x}$

Solution: $\sqrt[3]{54x^4}+4x\ \sqrt[3]{16x}=\sqrt[3]{27\cdot 2\cdot x^3\cdot x}+4x\ \sqrt[3]{8\cdot 2\cdot x}=\sqrt[3]{27}\sqrt[3]{x^3}\sqrt[3]{2\cdot x}+4x\ \sqrt[3]{8}\sqrt[3]{2\cdot x}=$

$$=3x\sqrt[3]{2x}+4x\cdot 2\sqrt[3]{2x}=3x\sqrt[3]{2x}+8x\sqrt[3]{2x}=\boxed{11x\sqrt[3]{2x}}$$

30. $\sqrt{45}-\sqrt{20}$

Solution: $\sqrt{45}-\sqrt{20}=\sqrt{5\cdot 9}-\sqrt{5\cdot 4}=\sqrt{5}\sqrt{9}-\sqrt{5}\sqrt{4}=3\sqrt{5}-2\sqrt{5}=\boxed{\sqrt{5}}$

31. $\left(4\sqrt{3}-3\sqrt{2}\right)\left(5\sqrt{3}+3\sqrt{2}\right)$

Solution: $\left(4\sqrt{3}-3\sqrt{2}\right)\left(5\sqrt{3}+3\sqrt{2}\right)=4\sqrt{3}\cdot 5\sqrt{3}+4\sqrt{3}\cdot 3\sqrt{2}-3\sqrt{2}\cdot 5\sqrt{3}-3\sqrt{2}\cdot 3\sqrt{2}=$

$$=20\left(\sqrt{3}\right)^2+12\sqrt{6}-15\sqrt{6}-9\left(\sqrt{2}\right)^2=20\cdot 3-3\sqrt{6}-9\cdot 2=60-18-3\sqrt{6}=\boxed{42-3\sqrt{6}}$$

32. Find the distance between the points (-5,8) and (1, 16).

Solution: The distance formula is: $d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$

$x_1=-5;\quad x_2=1;\quad y_1=8;\quad y_2=16$

$d=\sqrt{(-5-1)^2+(8-16)^2}$

Don't forget to use parentheses properly!!!

$d=\sqrt{(-6)^2+(-8)^2}$

$d=\sqrt{36+64}=\sqrt{100}=10$

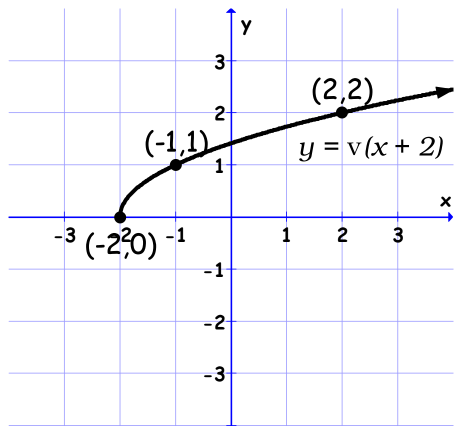
The answer is $\boxed{\text{10 units.}}$

Sketch a graph of each of the following functions. List the domain and range of the function. Label 3 exact points.

33. $f(x) = \sqrt{x+2}$

Solution: We always pick the first point to be at the corner of the graph (this is the x-value that makes the expression under the square root = 0. In this case this is x+2 = 0, or x =- 2. Then we pick two points from each side of the corner point and calculate the y values at them. Notice that the square root graph goes one way only so we will only receive y-values to one of the sides of this function. I picked x = 2, because it is easy to take root of 4. If I pick x = 0, or 1, then I'll have to take a root of 2 or 3, which are not exact numbers:

x	$f(x) = \sqrt{x+2}$
-4	$\sqrt{-4+2} = \sqrt{-2}$
-3	$\sqrt{-3+2} = \sqrt{-1}$
-2	$\sqrt{-2+2} = \sqrt{0} = 0$
-1	$\sqrt{-1+2} = \sqrt{1} = 1$
2	$\sqrt{2+2} = \sqrt{4} = 2$



The corner point is at (-2, 0).

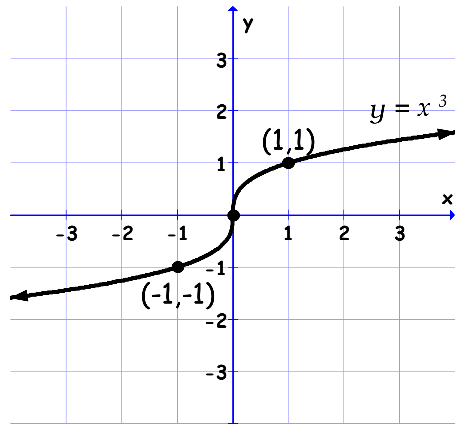
Domain: $[-2, \infty)$ (because we cannot plug values for x less than -2; if we do we get negative numbers under the root)

Range: $[0, \infty)$ (because there is no y-value lower than 0 on the graph; it is a closed interval, because there is a point that contains y=0.)

34. $g(x)=\sqrt[3]{x}$

Solution: We always pick the first point to be the x-value that makes the expression under the root = 0. In this case this is x = 0. Then we pick two points from each side of the vertex value and calculate the y values at them. If we work correctly the y values are going to be opposite numbers for each pair of x or the points are going to be symmetrical in respect of the origin.

x	$g(x)=\sqrt[3]{x}$
-8	$\sqrt[3]{-8}=-2$
-1	$\sqrt[3]{-1}=-1$
0	$\sqrt[3]{0}=0$
1	$\sqrt[3]{1}=1$
8	$\sqrt[3]{8}=2$



Domain: $(-\infty,\infty)$; Range: $(-\infty,\infty)$